are asked: How much mathematics is really indispensable for how much science? What does such indispensability indicate about the existence of mathematical objects? What really can be gleaned from Field's conservativeness results? And what non-trivial assumptions are required to carry out the modal applied mathematics programme? This is one of the best discussions on this rather neglected topic in the literature.

Altogether this is a deep and lucid discussion of an important foundational thesis. Certainly there are questions to be asked, particularly regarding the notion of possibility involved and to exactly what extent, if any, this is superior for arithmetic or analysis to other versions of structuralism like those of Resnik or Shapiro. Nevertheless it greatly furthers the structuralist case in the crucial intractable matter of set theory and raises new issues. For example, suppose the full power-set operation is dropped and we consider predicative set theory; can an analogue of the modal structuralist translations be obtained for the predicative theory, using some "reduced" version of second-order logic? PETER CLARK

JODY AZZOUNI. ***Metaphysical myths, mathematical practice. The ontology and epistemology of the exact sciences.*** Cambridge University Press, Cambridge, New York, and Oakleigh, Victoria, 1994, ix + 249 pp.

Metaphysical myths, mathematical practice is a very aptly named book, for in it Jody Azzouni tries to do two things, namely (i) present a comprehensive picture of mathematical theory and practice, and (ii) argue that if this picture is accurate, then the traditional metaphysical problems about the nature of mathematical objects are pseudo-problems. (Actually, the third and final part of the book does not fit so neatly into this characterization. In this portion of the book, Azzouni turns from ontological to epistemological concerns and, in particular, to the topic of a priority; he argues that while mathematical truths are not a priori in the traditional sense—i.e., are neither incorrigible nor independent of experience—they are epistemically special in that they can be derived in a formal system, and this gives them a certain sort of non-traditional a priority.)

As far as (i) is concerned, it seems to me that Azzouni's discussion of mathematical practice is not only (mostly) accurate, but also interesting and original. Azzouni's view, in a nutshell, is that "mathematics . . . is a collection of algorithmic systems," where a system is, basically, a set of sentences generated by a set of axioms and rules of inference; these systems are taken "to be fairly arbitrary in character; that is, there are no genuine constraints on what systems can look like" (p. 79). Now, by itself, this might not seem like a very original view, because it might seem like a version of formalism; but Azzouni's view also has some decidedly realistic (and, hence, anti-formalistic) threads woven into it; in particular, he thinks that mathematical singular terms refer and that mathematical systems—*all* mathematical systems, mind you—are true. But this is not a thoroughgoing realism, because, for Azzouni, "the truth predicate is just a device for asserting groups of sentences without . . . listing them" (p. 147), and so mathematical truth turns out to be conventional in a certain way, although, Azzouni argues, not in a way that leaves him susceptible to Quinean attack. (It might seem that taking all systems to be true leads to inconsistency, since two different systems can contradict one another; but this can be avoided by giving different terminologies to conflicting systems, e.g. using 'set' in ZFC and 'set*' in ZF+ ~C, so that their terms do not co-refer. Now, that is not to say that, on Azzouni's view, two systems *cannot* co-refer; he thinks they can, and he thinks that the question of which systems co-refer is a matter of stipulation.)

The best thing about Azzouni's discussion of mathematical practice is that he brings to our attention some very interesting features of mathematical practice that have, until now, been largely ignored in the philosophical literature. For example, there is a long, thought-provoking discussion near the beginning of the book about the sorts of errors that are possible in mathematics and how certain sorts of errors that we make with empirical terms simply cannot happen with mathematical terms. For example, Azzouni points out that whereas we can mistake John for his twin brother James, we cannot mistake 4 for 3: we can mistakenly think that '4' is the name of 3, and we can err in calculation and think that, for example, '24/6' refers to 3, but in neither case do we confuse the two numbers 4 and 3. The point of these discussions is twofold. First, they introduce various new puzzles for the philosophy of mathematics, for they bring to our attention certain (previously unnoticed) facts about mathematical practice that need to be explained. And secondly, they shed new light on the old puzzles that philosophers of mathematics have traditionally been concerned with; for example, Azzouni's discussion makes it clear that we ought not to try to solve the (Benacerrafian) problem of knowledge—i.e., the problem of explaining how we can acquire knowledge of mathematical objects—by coming up with an *epistemic role* for mathematical

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objects to play in the acquisition of such knowledge, because if we look at mathematical practice, it is clear that mathematical objects simply do not play any such role.

This brings me to (ii), that is, to the topic of how Azzouni tries to argue that the traditional metaphysical problems about the nature of mathematical objects are pseudo-problems. The most important of these problems is the problem of knowledge just mentioned. Azzouni's claim is that once we have an accurate understanding of mathematical practice—in particular, once we notice how metaphysically *thin* mathematical truth and existence are—it becomes clear that there simply is not a problem here. According to Azzouni, all mathematical systems are true and the objects of all such systems exist; there is nothing more one needs for mathematical truth and existence than the introduction of a collection of axioms and rules of inference. Because of this, knowledge of mathematical objects is a very cheap commodity: whenever we develop an algorithmic system, we get mathematical knowledge. And, therefore, philosophical worries about how knowledge is possible are simply misplaced; they are born of an inaccurate conception of mathematical practice.

It is worth looking at all of this in connection with the issue of mathematical Platonism, since the problem of knowledge is so intimately connected to that issue, and since the question of whether Azzouni is a Platonist brings to light another interesting facet of his view. At the end of the book, Azzouni claims that his view is somewhere between Platonism and anti-Platonism, because it endorses some of the tenets of both of these views but all of the tenets of neither. This, I think, is right. But if we concentrate *solely* on the issue of whether there are mathematical objects that exist "out there" (i.e., independently of us and outside spacetime), letting 'Platonism' denote the view that answers this question in the affirmative, then Azzouni seems to go anti-Platonist; he tells us on the very last page that "mathematical objects *are* posits, and posits are not, strictly speaking, independent of their positors." It seems to me, however, that this stance is not entailed by the view he develops throughout the book; that view, it seems, is simply neutral on the question of whether there are mathematical objects that exist outside spacetime.

I think there is something deeply right about this neutrality, for I do not think that the totality of facts about mathematical practice determines whether Platonism or anti-Platonism is true. Thus, I think there is something right about Azzouni's attitude toward the problem of knowledge. He has shown that in describing what mathematicians do, the question of whether mathematical objects exist outside spacetime need not arise at all. But the problem of knowledge only *becomes a problem* when we take a stand on that question—in particular, when we assert that mathematical objects *do* exist outside spacetime, thus leaving ourselves open to the question, 'How on earth could we acquire knowledge of things to which we have no epistemic access?' Thus, Azzouni has shown that if we are careful, we can provide an accurate description of mathematical practice without ever encountering the problem of knowledge.

But despite the fact that I think Azzouni is right about this, I do not agree that he has shown the problem of knowledge to be a pseudo-problem, and more generally, I do not think he has done away with the metaphysical issues surrounding mathematics. What Azzouni has shown is that *if* your task is the hermeneutical task of providing an accurate picture of mathematical practice, *then* you need not address the problem of knowledge. But one might have a different task in mind, viz., the metaphysical task of ascertaining whether Platonism is true, that is, whether there are mathematical objects which exist independently of us and outside spacetime.

Let us assume that this is our task. And let us assume that Azzouni's view of mathematical practice is, for the most part, right. This means that there is nothing in the practice of mathematics, that is, in what mathematicians do, that answers our metaphysical question. Nonetheless, there either *are* objects "out there," answering to our mathematical theories, or else there are *not*—actually, it may be that there is no fact of the matter here, but let us leave that worry to one side—and we can try to settle this question via philosophical argument. Moreover, it turns out that while mathematical practice by *itself* does not answer our question, facts about mathematical practice can be used as premisses in arguments for and against Platonism. (Whether any of these arguments are actually sound is another matter.) For instance, one argument for Platonism is based upon the undeniable fact that mathematics is applicable to empirical science: the claim here is that anti-Platonism is inconsistent with this fact, that is, that in order to account for the fact of applicability, we need to presume that (a) mathematics is true and (b) this truth involves there being mathematical objects that exist independently of us. Now, this argument might seem to fly in the face of one of Azzouni's observations about mathematical practice, viz., that there is something very cheap about mathematical truth; but I do not think that this is necessarily the case, because one might think that Platonists can simply explain *why* mathematical truth is so easily attained, despite the fact that it consists in the accurate characterization of objectively existing mathematical objects. Now, of course, one might also doubt that Platonists can account for the cheapness of mathematical truth, and so this generates an argument on the other side of our debate, that is, an argument *against* Platonism that is based upon a fact about mathematical practice.

Another argument against Platonism that has this same general form is what I have been calling the problem of knowledge; the claim here is that Platonism is inconsistent with the fact that mathematicians have mathematical knowledge; or to make the argument more powerful, anti-Platonists can claim that Platonism is inconsistent with Azzouni's observations that (a) mathematical knowledge is very cheap—whenever we introduce a system, we get knowledge—and (b) mathematical objects do not play any role in the acquisition of mathematical knowledge. The idea here, of course, is that if mathematical objects really did exist independently of us and outside spacetime, then knowledge of such objects simply would not be cheap (and the objects would have to play some epistemic role). But, of course, Platonists might disagree: they might think it possible to explain *why* it is so easy to obtain knowledge of objects that exist independently of us and outside spacetime and that play no role in our acquisition of knowledge of them.

So I do not think that the problem of knowledge is a pseudo-problem; it is a real problem for anyone inclined to adopt mathematical Platonism. More generally, I do not think that Azzouni has done away with the metaphysical issues surrounding mathematics, because *even if we endorse (most of) his picture of mathematical practice*, we might *still* want to endorse Platonism (and take on the problem of knowledge) or anti-Platonism (and take on the problem of applicability). But I would like to emphasize that despite my disagreement on this point, I think that Azzouni's book is interesting, important, and well worth reading.

One more point. I have said that the view Azzouni develops is neutral on the question of whether mathematical objects exist "out there," but that, at the end of the book, he takes a stand on this question and goes anti-Platonist. If this is right, then he has entered the metaphysical arena, and so, being an *anti*-Platonist, he owes an account of applicability. So I should point out that he does say a few words on this topic. He begins with a very interesting discussion of how the success of applied mathematics can be likened to the success of certain species of animals. He then turns to the more important issue of saying why an appeal to mathematical objects is not needed to explain applicability; in this connection, he claims that we can account for applicability by appealing to properties of the *physical* objects to which the mathematics is being applied. His discussion here is extremely brief—far too brief to constitute anything like a complete answer to the problem of applicability—but I should say that it does seem promising and intuitively pleasing.

MARK BALAGUER

ANIL GUPTA and NUEL BELNAP. ***The revision theory of truth.*** Bradford books. The MIT Press, Cambridge, Mass., and London, 1993, xii + 299 pp.

Despite Tarski's analysis of the problems inherent in defining truth and his two-language solution, there has remained a feeling that we nonetheless ought to be able to give some coherent account of how the notion of truth for a language functions within the language itself. People use the concept of truth effectively in ordinary discourse all the time, so it should be possible to incorporate at least some aspects of that use into a formal language without paradox. In the mid-1970's and early 1980's, various *ad hoc* discussions gave way to two systematic approaches: fixed-point theory and revision theory. This book is a very thorough account of the latter theory by two of its principal creators.

The fundamental intuition behind revision theory as it is presented here is stated succinctly in the Preface: "Truth is a circular concept." This leaves the authors with two tasks: to explain how one can usefully make sense of a concept with a circular definition, and then to indicate how truth falls under such a framework. Their solution to the latter is by far the simpler, although novel and undoubtedly controversial. The authors argue that the Tarski biconditionals should be read not as material biconditionals but rather as a definitional scheme: $(*)$ 'A' is true *by definition* means A. The circularity arises, of course, when 'A' itself contains an occurrence of the term 'true.' In order to solve the problems of circularity, the authors suggest that any circular definition can be interpreted as a *rule* for taking a hypothesis about the possible extension of the concept being defined and revising it to create a new, and possibly better, candidate for that extension. In the present context an initial hypothesis about the extension of the truth predicate can be used in the right-hand side of $(*)$ to create a new extension specified by the left-hand side.